JOHN CURTIN
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 3, 2018
(Vectors and Geometric proofs)

Section One: Calculator Free

Student's Name: _Solutions_

Total Marks: 16
Time Allowed: 15 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST
Standard Items: Pens, pencils, eraser, ruler

Special Items: WACE Formula Sheet

INSTRUCTIONS TO STUDENTS
Do not open this paper until instructed to do so.
You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

Question 1 [2 marks]

Find the scalar projection in the direction 46° of a vector of magnitude 20 in the direction 166°.
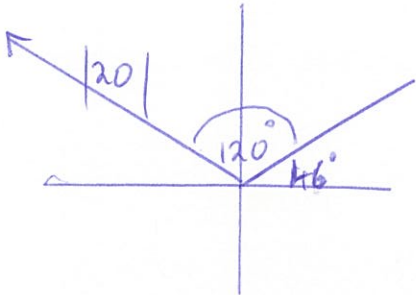
$|20| \cdot \cos 120^\circ$ ✓

$= -10$ ✓

Question 2 [5 marks]

Ship A has position vector $2\mathbf{i} + 4\mathbf{j}$. Ships B and C are such that the position vector of B relative to C is $7\mathbf{i} - 4\mathbf{j}$ and the position vector of C relative to A is $-4\mathbf{i} - \mathbf{j}$. Find the position vector of ship B.

$_B\underline{V}_C = 7i - 4j = \underline{V}_B - \underline{V}_C$ ✓

$_C\underline{V}_A = -4i - j = \underline{V}_C - \underline{V}_a$ ✓

$\underline{V}_A = 2i + 4j$

$\underline{V}_B = {}_B\underline{V}_C + \underline{V}_C$ ✓

$= {}_B\underline{V}_C + {}_C\underline{V}_A + \underline{V}_A$ ✓

$= \begin{pmatrix} 7 \\ -4 \end{pmatrix} + \begin{pmatrix} -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\therefore \quad \underline{V}_B = 5i - j$ ✓

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Question 3 [4 marks = 2, 2]

If **a** = 2**i** -3**j** and **b** = 7**i** + x**j** are two vectors, find:

(a) *x* if **a** and **b** are parallel.

$$\binom{2}{-3} = K \binom{7}{x} \Rightarrow K = \frac{2}{7} \checkmark$$

$$x = -3 \times \frac{7}{2}$$

$$= \frac{-21}{2} \checkmark$$

(b) *x* if **a** and **b** are perpendicular.

$$\binom{2}{-3} \cdot \binom{7}{x} = 0 \checkmark$$

$$14 + -3x = 0$$

$$x = \frac{14}{3} \checkmark$$

Question 4 [5 marks = 2, 3]

Given **u**=2**i** – 3**j**, **v**=5**i** + 6**j** and **w**= -2**i** + 4**j** find:

(a) **u.w**

$$\binom{2}{-3} \cdot \binom{-2}{4} = -4 + -12 \checkmark$$

$$= -16 \checkmark$$

(b) **u.(2v – 3w)**

$$\binom{2}{-3} \cdot \left[2\binom{5}{6} - 3\binom{-2}{4} \right] \checkmark$$

$$\binom{2}{-3} \cdot \binom{10 + 6}{12 - 12}$$

$$= \binom{2}{-3}\binom{16}{0} \checkmark$$

$$= 32 \checkmark$$

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TEST 3, 2018
(Vectors and Geometric proofs)

Section Two: Calculator Assumed

Student's Name: _Solutions_

Total Marks: 23
Time Allowed: 25 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Up to three approved calculators
One page (unfolded A4 sheet) front and back of Notes
WACE Formula Sheet

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Question 5 [6 marks = 1, 5]

A car is travelling due west at 80 km/hr. The passengers of the car observe that a train appears to be travelling due south at 60 km/hr.

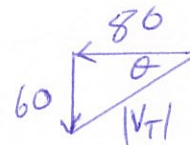(a) Draw a diagram representing this situation labelling all vectors clearly.

$$V_c = -80i$$

$$TV_c = -60j$$

$$V_T$$
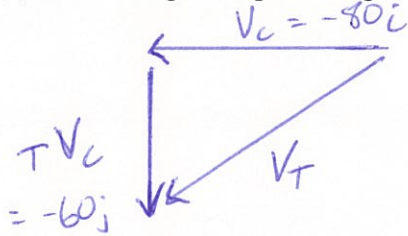
(b) Calculate the magnitude and bearing of the true velocity of the train.

$$TV_c = \begin{pmatrix} 0 \\ -60 \end{pmatrix} = V_T - V_L \checkmark$$

$$\begin{pmatrix} 0 \\ -60 \end{pmatrix} = V_T - \begin{pmatrix} -80 \\ 0 \end{pmatrix}$$

$$V_T = \begin{pmatrix} -80 \\ -60 \end{pmatrix} \checkmark$$

$$|V_T| = 100 \checkmark , \quad \theta = 36.87° \checkmark$$

$$\therefore \text{ Bearing} = 270° - 36.87 = 233.13° \checkmark$$

Question 6 [5 marks]

Consider the points A (-1, 6), B (-3, -2) and C (7, 3). Calculate the angle between **BA** and **BC**.

$$\vec{BA} = \vec{OA} - \vec{OB} = \begin{pmatrix} 2 \\ 8 \end{pmatrix} \checkmark \qquad \vec{BC} = \vec{OC} - \vec{OB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \checkmark$$

$$\cos\theta = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|} \checkmark$$

$$= \frac{\begin{pmatrix} 2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 5 \end{pmatrix}}{\sqrt{2^2+8^2} \ \sqrt{10^2+5^2}}$$

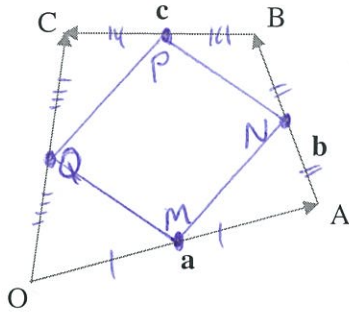$$\approx 0.65 \checkmark$$

$$\theta = 49.4° \checkmark$$

$$\boxed{11}$$

Question 7 [6 marks]

Use vector geometry and the diagram below to prove that the midpoints of the sides of <u>**any**</u> <u>**quadrilateral**</u> join to form a parallelogram. (Let OA = **a**, AB = **b**, BC = **c**)

$$\overrightarrow{MN} = \tfrac{1}{2}\underline{a} + \tfrac{1}{2}\underline{b} \quad \checkmark$$

$$\overrightarrow{NP} = \tfrac{1}{2}\underline{b} + \tfrac{1}{2}\underline{c} \quad \checkmark$$

$$\overrightarrow{OC} = \underline{a} + \underline{b} + \underline{c} \quad \checkmark$$

$$\overrightarrow{QP} = \tfrac{1}{2}(\underline{a}+\underline{b}+\underline{c}) - \tfrac{1}{2}\underline{c}$$
$$= \tfrac{1}{2}\underline{a} + \tfrac{1}{2}\underline{b} \quad \checkmark$$

$$\overrightarrow{MQ} = \tfrac{1}{2}(\underline{a}+\underline{b}+\underline{c}) - \tfrac{1}{2}\underline{a}$$
$$= \tfrac{1}{2}\underline{b} + \tfrac{1}{2}\underline{c} \quad \checkmark$$

$$\therefore \overrightarrow{MN} = \overrightarrow{QP}$$
$$\text{and } \overrightarrow{MQ} = \overrightarrow{NP} \quad \checkmark$$

$$\therefore \quad MNPQ \text{ forms a parallelogram}$$

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Question 8 [6 marks]

For $\triangle OAB$, the point C is the midpoint of side AB. Prove that: $2OC^2 + 2AC^2 = OA^2 + OB^2$.
(Let OA = **a** and OB = **b**)
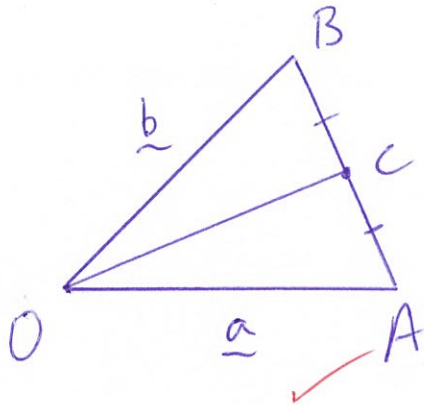
$$\vec{AB} = \underline{b} - \underline{a}$$

$$\vec{AC} = \tfrac{1}{2}(\underline{b} - \underline{a}) \checkmark$$

$$\vec{OC} = \vec{OA} + \vec{AC}$$

$$= \underline{a} + \tfrac{1}{2}(\underline{b} - \underline{a})$$

$$= \tfrac{1}{2}(\underline{a} + \underline{b}) \checkmark$$

$$LHS = 2(\vec{OC}^2 + 2\vec{AC}^2)$$

$$= 2\left(\tfrac{1}{4}(\underline{a} + b)^2 + \tfrac{1}{4}(\underline{b} - \underline{a})^2\right) \checkmark$$

$$= \tfrac{1}{2}\left((\underline{a} + b)^2 + (\underline{b} - \underline{a})^2\right)$$

$$= \tfrac{1}{2}\left(\underline{a}^2 + 2\underline{a}\underline{b} + b^2 + b^2 - 2\underline{b}\underline{a} + \underline{a}^2\right) \checkmark$$

$$= \tfrac{1}{2}(2\underline{a}^2 + 2b^2)$$

$$= \underline{a}^2 + \underline{b}^2$$

$$= OA^2 + OB^2 \checkmark$$

$$= RHS$$